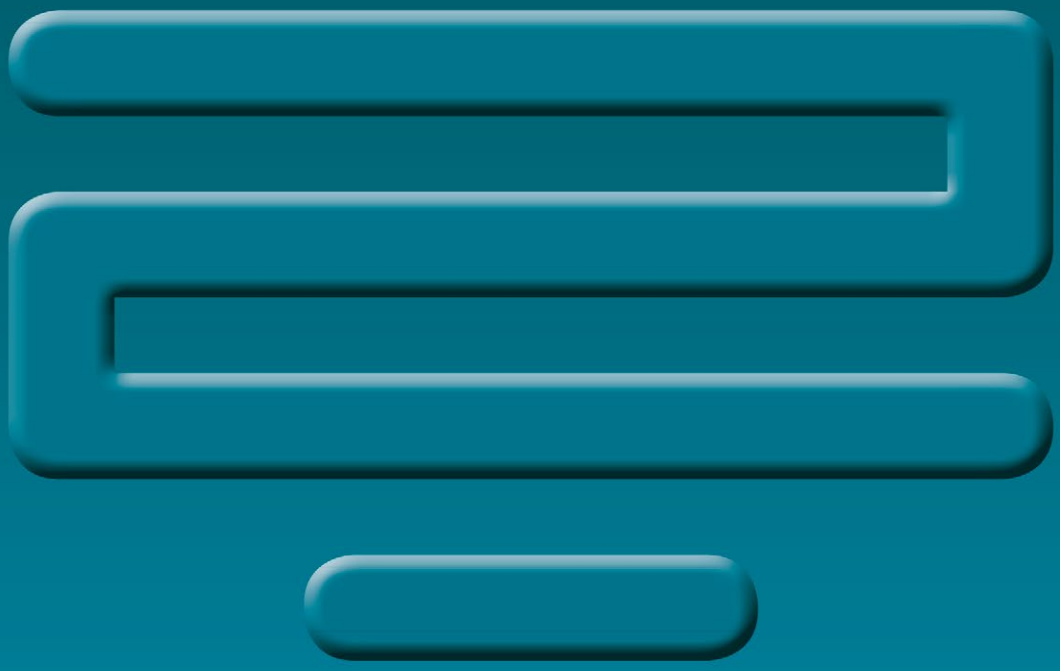
INFERENCES WITH IGNORANCE: LOGICS OF QUESTIONS

INFERENTIAL EROTETIC LOGIC & EROTETIC EPISTEMIC LOGIC



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Inferences with Ignorance: Logics of Questions
Inferential Erotetic Logic & Erotetic Epistemic Logic

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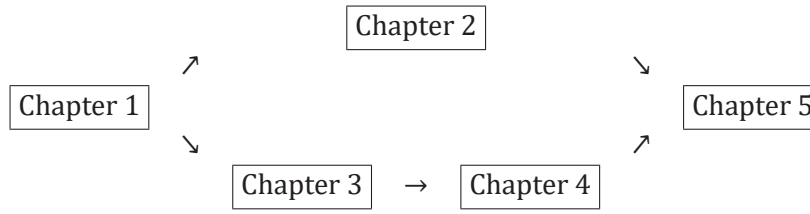
PREFACE

As the title indicates, the book is about logic of questions (erotetic logic) which is a branch of non-classical logic. The text goes in for two formal (logical) systems of questions with the important role of erotetic inferential structures. The first one, *inferential erotetic logic*, had been originally developed in Poland in 1990s. I used this system in a slightly rearranged version that serves as the main inspiration for erotetic logic studied in the epistemic framework. The second system, *erotetic epistemic logic*, is just such a combination of epistemic logic with questions that is open for application in public announcement logic.

The text is based on my doctoral dissertation [36] which was finished during the year 2010. I decided to change some parts with respect to reviews and discussions with my colleagues and students. Nonetheless, the structure and main results are almost the same. I tried to incorporate all objections as well as the advice of my referees or, at least, I comment on them in the last chapter. The aim of this book is rather different from the thesis, therefore I checked allof the proofs and explanations and amended them. Although I believe that the text is more comprehensive now, some background knowledge of the reader is expected. A knowledge of elementary notions from formal (mathematical) logic and a basic knowledge of modal logic is assumed, and would be very helpful in reading the part starting at Chapter 3. Nice introductions to (modal) epistemic logic are introductory chapters in [48] and [11].

STRUCTURE OF THE BOOK

The book includes two main parts that can be read independently. The first one is Chapter 2 and the second one consists of Chapters 3 and 4. Chapter 1 serves as an introduction and provides common a methodology for both parts. The last Chapter 5 contains some final remarks on the methodology used, summarizes the main results, problems, related approaches, and also further directions of the branch.



If we compare the contents of the chapters, we may find another division of the book. Both Chapter 2 and Chapter 3 can be understood as a study of the 'logic of questions'. There we are interested in basic erotetic structures,

i.e., inferences with questions, relationships of questions and declaratives, and answerhood conditions. However, Chapter 4 introduces questions as a part of communication; a dynamic approach is applied in the erotetic epistemic framework introduced.[1]

CHAPTER 1: LOGIC AND QUESTIONS

The chapter introduces a multi-paradigmatic situation in the methodology of erotetic logic and contains a short historical overview of this branch with a special emphasis on recent developments.[2] We briefly introduce *inferential erotetic logic*, Groenendijk-Stokhof's intensional approach, and some developments of these theories. However, the core of the chapter is devoted to a formalization of questions based on sets of answers. We justify the usefulness of the set-of-answers methodology in the study of erotetic consequence relations as well as in an epistemic interpretation of questions.

This chapter is based on the paper [37].

CHAPTER 2: CONSEQUENCE RELATIONS IN INFERENTIAL EROTETIC LOGIC

This part is aimed at the studying of relationships among consequence relations that are introduced in *inferential erotetic logic* (IEL). We keep the framework of IEL, but the question representation uses the methodology from Chapter 1. IEL requires that declarative and interrogative formulas are not mixed on the object-language level; answers are strictly declarative sentences. The defined consequence relations with questions are naturally based on the multiple-conclusion entailment among sets of declarative formulas. We add the term *semantic range* of a question to the terminology of IEL and work with sets of declaratives as associated with classes of models. This 'model-based approach' makes proofs and properties very transparent. The chapter is a technical overview of some IEL concepts and their properties. We understand the IEL presented as providing a general framework and inspiration for the work with inferences among questions and declaratives.

Chapter 2 can be read as a full introduction to the topic; no reading of another text is required. The chapter was originally published in [35].

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1 Dynamic-like approaches bear the name 'logic of inquiry' in literature.

2 Approximately till the year 2010. Comments on the newest literature are in Chapter 5.

CHAPTER 3: EPISTEMIC LOGIC WITH QUESTIONS

The main goal here is to incorporate questions in a general epistemic framework. Questions are represented by finite sets of direct answers and their 'satisfiability' in a state of an epistemic model is based on three conditions that express ignorance and the presuppositions of a questioner.[3] In the framework of normal modal logic, a question becomes a complex modal formula. Inspired by inferential structures in IEL, we show that there are 'philosophically' similar structures based on a classical implication. The rest of the chapter is devoted to answerhood conditions and the role of the implication with respect to the epistemic context and conjunctions of yes-no questions.

This chapter cannot be read without a basic knowledge of modal (epistemic) logic.

CHAPTER 4: A STEP TOWARDS THE DYNAMIZATION OF EROTETIC LOGIC

This chapter takes full advantage of the multi-agent extension of the setting from Chapter 3 and can be considered an application of the introduced erotetic-epistemic approach in a dynamic framework. We define here *public announcement logic* based on the S5 modal system extended by group modalities. The askablity of questions as well as answerhood conditions are studied from the viewpoint of groups of agents. As an application, we show the role of questions and group modalities in a (communication-like) answer 'mining' in a group of agents.

Some results of the chapter were published in [38, 39].

CHAPTER 5: CONCLUSION

The last chapter includes a brief overview of related works, main results, possibilities of future directions, and comments on the problematic parts of our approach. This chapter is the result of many discussions. The important part of the subsection devoted to weak points and problems is based on the reviews to the original doctoral thesis. A list of up-to-date publications cannot be complete. However, we tried to add important papers closely related to our approach.

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3 We will use the term *askability* of a question (for an agent in a state).